The determination of earth's gravity field model by torus approach with GOCE data

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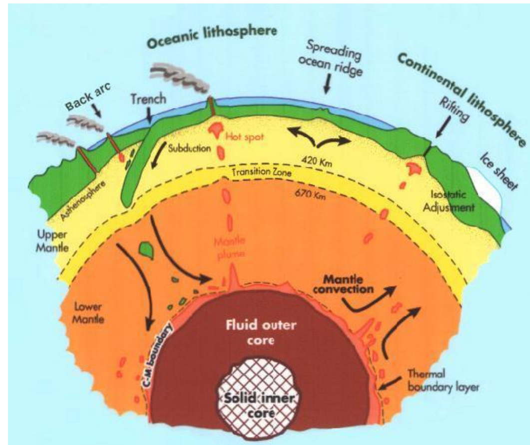

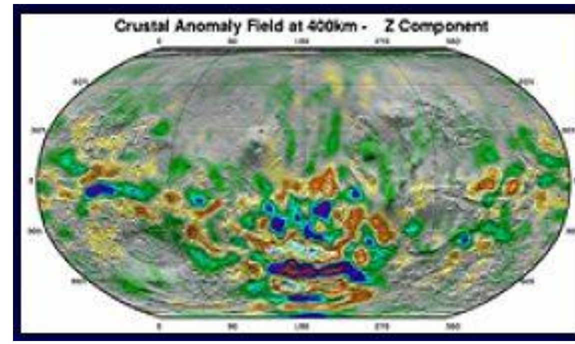

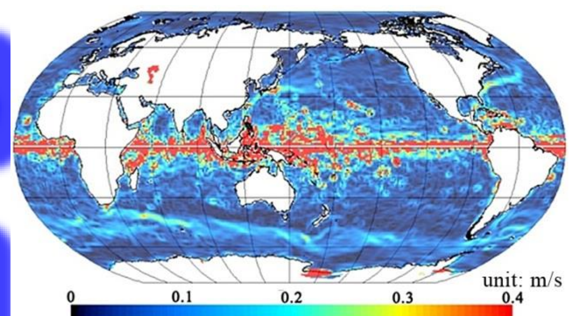

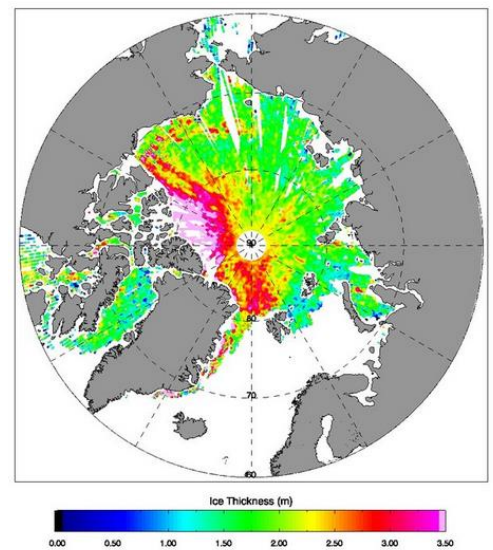

Introduction: significance of earth's gravity field

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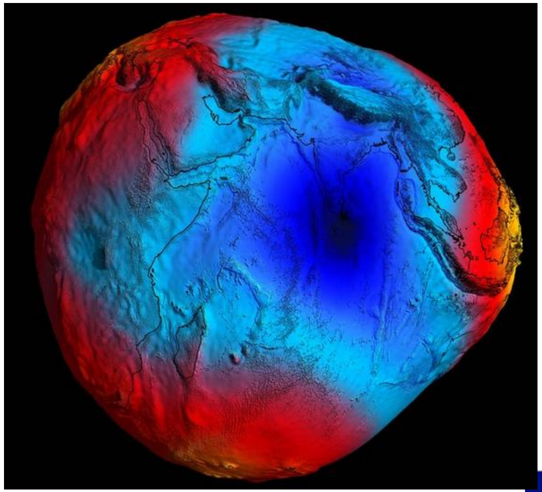
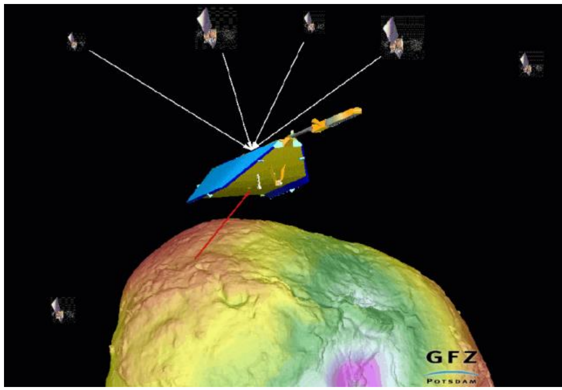
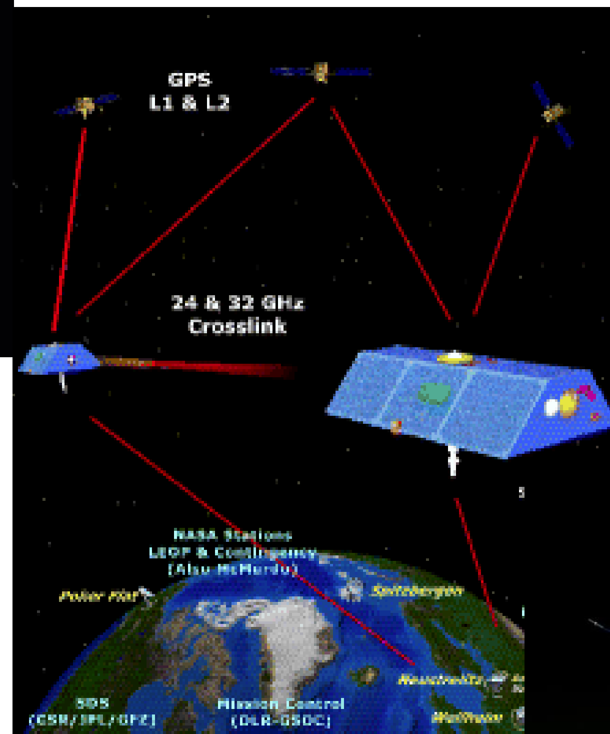


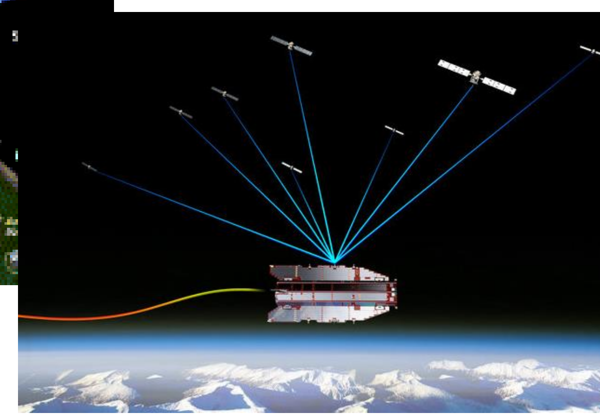
Introduction: Gravity Field Exploration Missions



CHAMP



GRACE

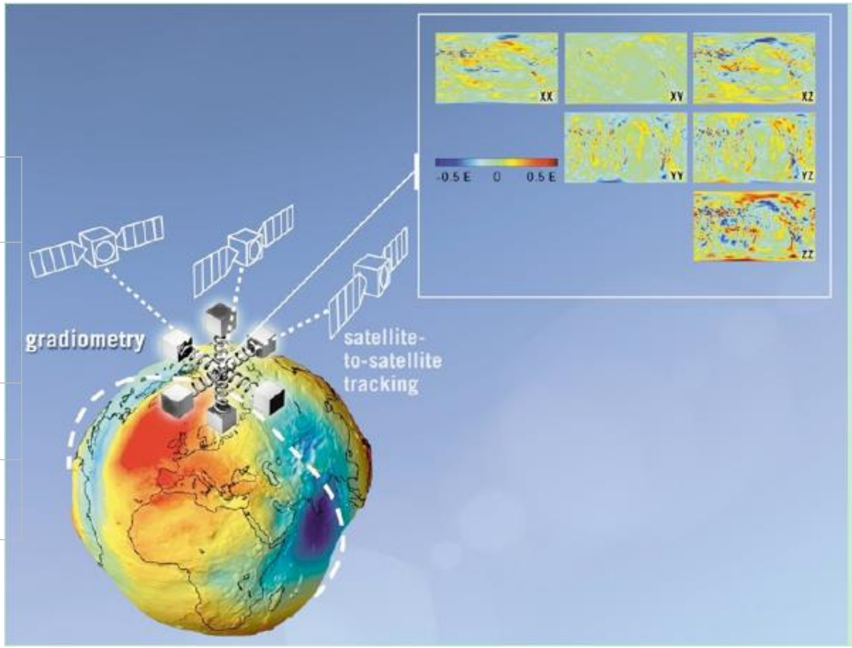


GOCE

Introduction: GOCE Mission

GOCE overview

role	Earth observation (EO)
orbit	Sun-synchronous ~224 km
Launch date	17 March 2009
Complete	11 Nov 2013



Mission objectives

- ✓ determine gravity-field anomalies with an accuracy of 1 mGal.
- ✓ determine the geoid with an accuracy of 1-2 cm.
- ✓ achieve the above at a spatial resolution better than 100 km.

Introduction: GOCE gravity field modeling

There are several different approaches applied to recover the GOCE gravity field.

Direct

time-wise

space-wise

SA

Tensor invariant

Rosborough

…

Introduction: GOCE gravity field modeling

There are several different approaches applied to recover the GOCE gravity field.

Direct
time-wise
space-wise

Approaches used to determine GOCE gravity filed models by the HPF

SA
Tensor invariant method
Rosborough
…

Introduction: GOCE gravity field modeling

the max d/o is 200 → **40397 geopotential coefficients to be determined**

tens of millions of SGG observations

Forming the normal equation and inverting the normal matrix will demand huge computation resources, which could not be realized using single processors.

Introduction: GOCE gravity field modeling

Torus approach

✓combines the properties of space-wise and time-wise methods

✓using the 2D-FFT and the block-diagonal least-squares adjustment

This method has been successfully applied to simulated data, but not used to compile the gravity field model with the real GOCE observitions.

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Torus approach

Representation on the Sphere

$$V_{ij}(r,\theta,\lambda)=\sum_{m=0}^{\infty}\left[A_m^{ij}(r,\theta)\cos m\lambda+B_m^{ij}(r,\theta)\sin m\lambda\right]$$

Representation along the Orbit

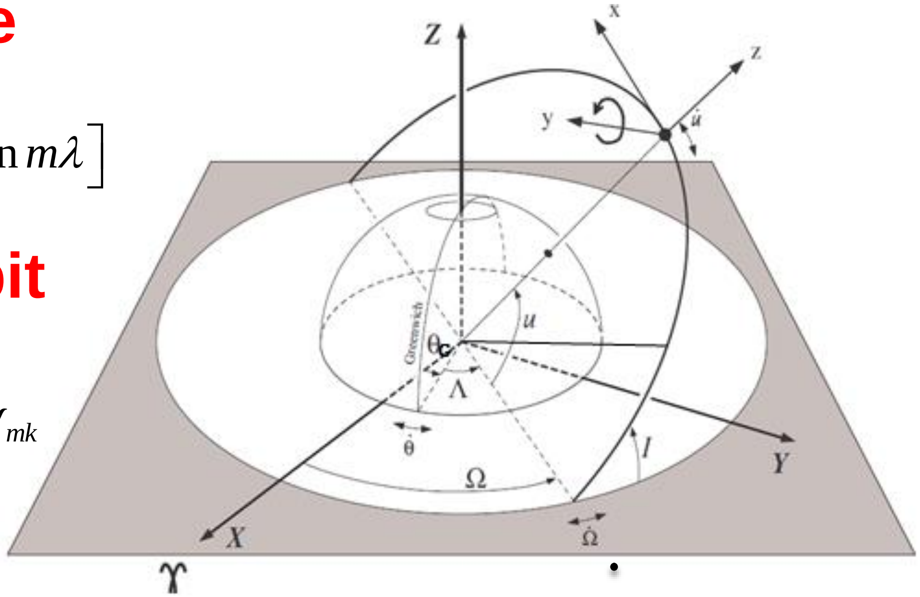
$$V_{ij}(r,I,u,\Lambda)=\sum_{m=0}^{N}\sum_{k=-N}^{N}A_{mk}^{ij}\cos\psi_{mk}+B_{mk}^{ij}\sin\psi_{mk}$$

$$\begin{Bmatrix}A_{mk}^{ij}\\ B_{mk}^{ij}\end{Bmatrix}=\sum_{n=n_{\min}[2]}^{N}H_{nmk}^{ij}(r,I)\begin{Bmatrix}\alpha_{nm}^{ij}\\ \beta_{nm}^{ij}\end{Bmatrix}$$

$$\psi_{mk}=ku+m\Lambda$$

$$u=u_0+\dot{u}\Delta t$$

$$\Lambda=\Lambda_0+\dot{\Lambda}\Delta t$$

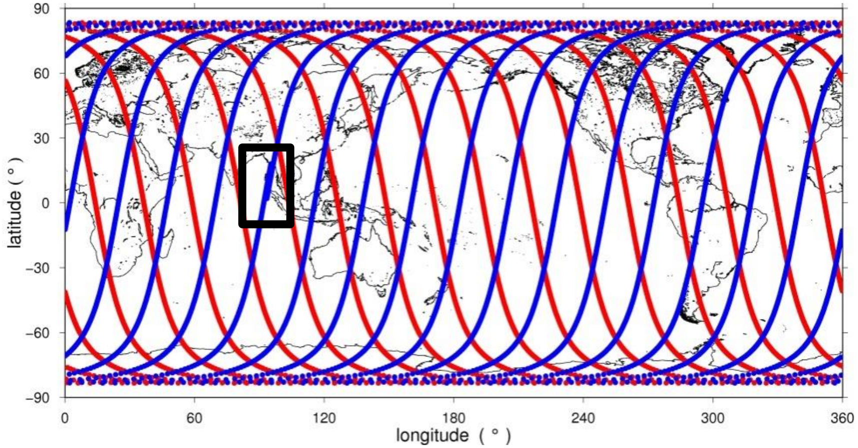


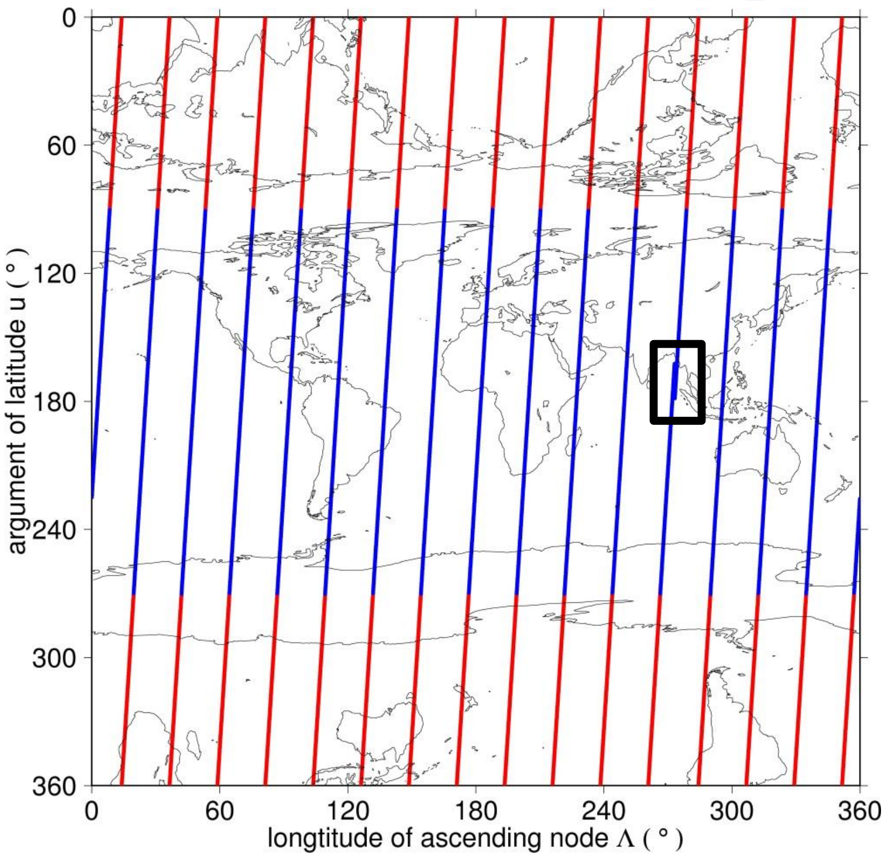
Orbit configuration

u is the argument of latitude

Λ is the longitude of ascending node

Torus approach

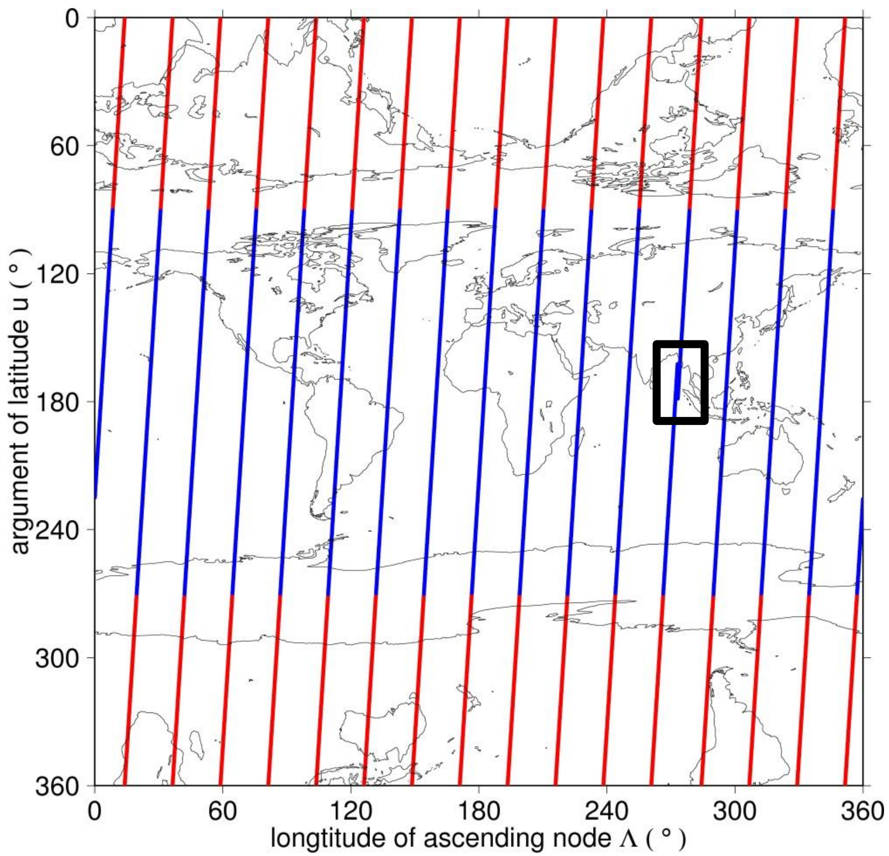


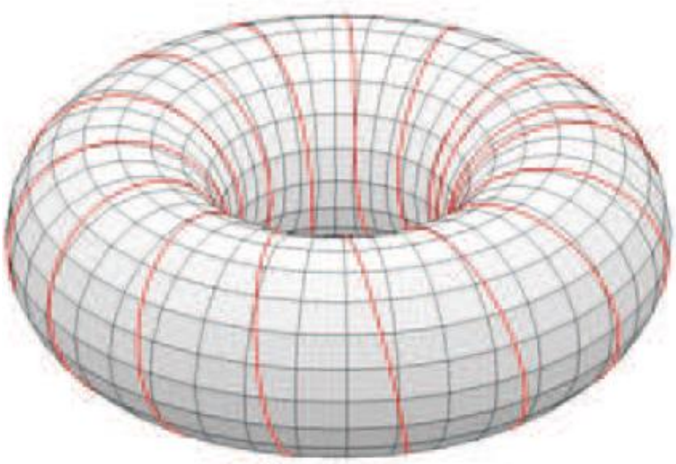


GOCE satellite orbits for one day（the red are ascending arcs，the blue are descending arcs）

Deutscher Akademischer Austausch Dienst
German Academic Exchange Service

Torus approach





Orbits on Torus

Torus approach

$$V_{ij}\left(r,I,u,\Lambda\right)=\sum_{m=0}^{N}\sum_{k=-N}^{N}A_{mk}^{ij}\cos\psi_{mk}+B_{mk}^{ij}\sin\psi_{mk}$$

$$\left\{\begin{matrix}A_{mk}^{ij}\\B_{mk}^{ij}\end{matrix}\right.=\sum_{n=n_{\min}[2]}^{N}H_{nmk}^{ij}\left(r,I\right)\left\{\begin{matrix}\alpha_{nm}^{ij}\\\beta_{nm}^{ij}\end{matrix}\right.$$

$$\psi_{mk}=ku+m\Lambda$$

$$u=u_0+\dot{u}\Delta t \qquad \Lambda=\Lambda_0+\dot{\Lambda}\Delta t$$

a nominal orbit

$$V_{ij}\left(u,\Lambda\right)=\sum_{m=0}^{N}\sum_{k=-N}^{N}A_{mk}^{ij}\cos\psi_{mk}+B_{mk}^{ij}\sin\psi_{mk}$$

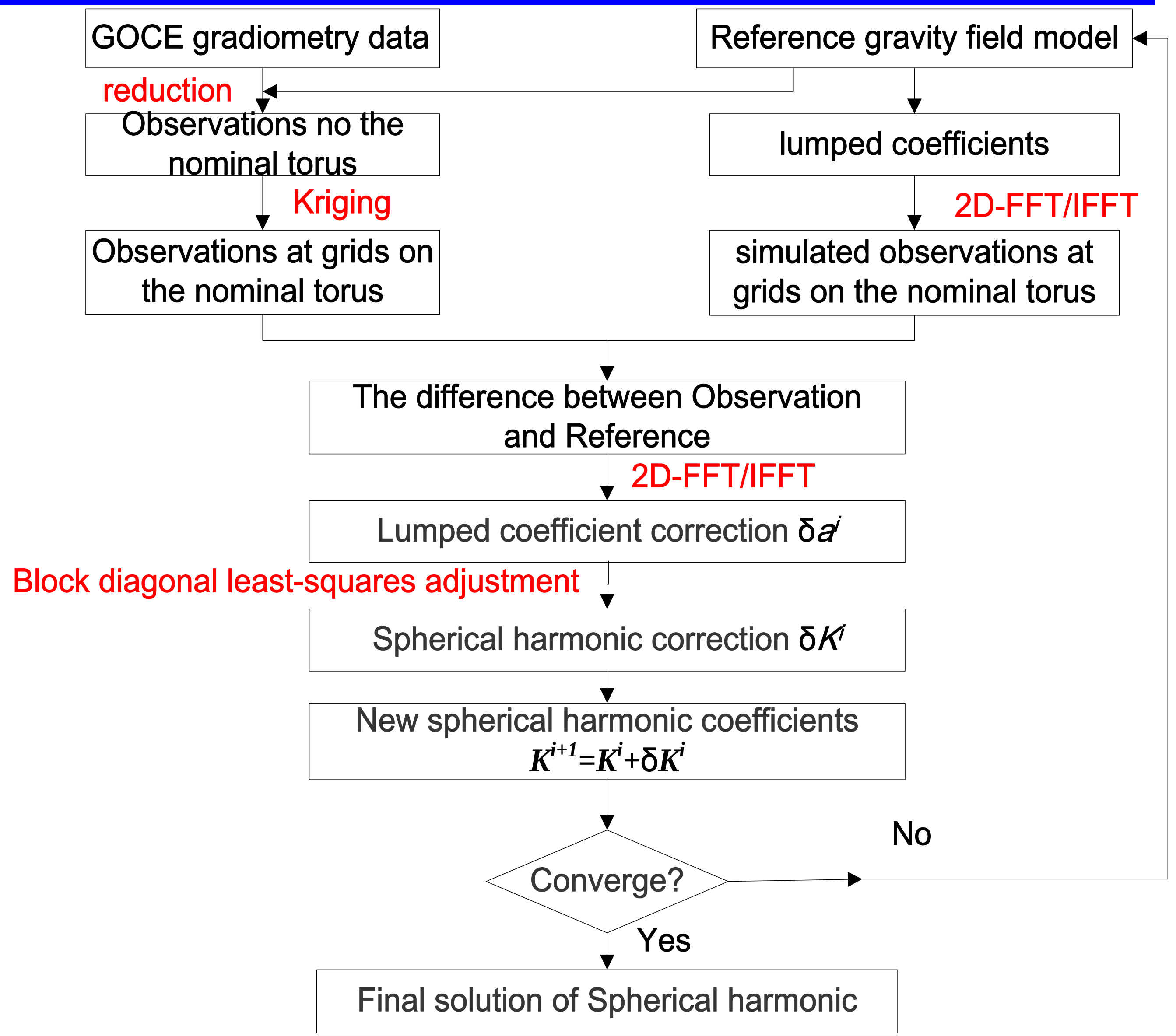
2D Fourier series

$$V_{xx}:H_{lmk}^{xx}=\frac{GM}{R^3}\left(\frac{R}{r}\right)^{l+3}\left[-\left(k^2+l+1\right)\right]\bar{F}_{lmk}\left(I\right)$$

$$V_{yy}:H_{lmk}^{yy}=\frac{GM}{R^3}\left(\frac{R}{r}\right)^{l+3}\left[k^2-\left(l+1\right)^2\right]\bar{F}_{lmk}\left(I\right)$$

$$V_{zz}:H_{lmk}^{zz}=\frac{GM}{R^3}\left(\frac{R}{r}\right)^{l+3}\left[\left(l+1\right)\left(l+2\right)\right]\bar{F}_{lmk}\left(I\right)$$

Torus approach-procedures



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- Torus with real GOCE gradiometry data
- Conclusions and outlooks

Torus with simulated satellite gradiometry data

✓ **Orbit Data**

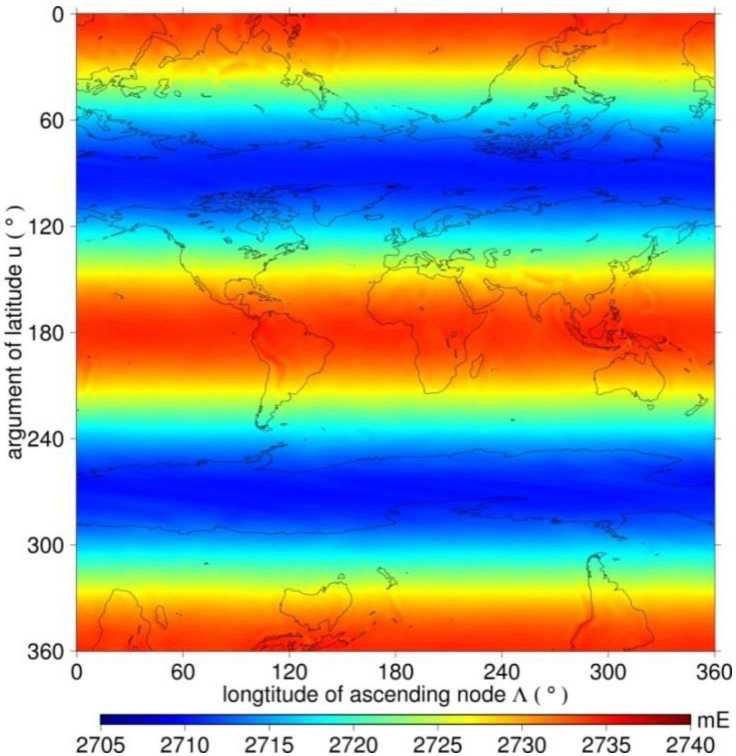
SST_PRD_2(2009.11.1~12.31, 61days), sampling interval is 10s.

✓ **gradiometry data (only *Vzz*)**

observations are simulated on GOCE real orbits using the model EGM2008, the max d/o are 200.

✓ **reference model: EGM96**

✓ **white noise 5mE/Hz$^{1/2}$**



Simulated V_{zz} observations

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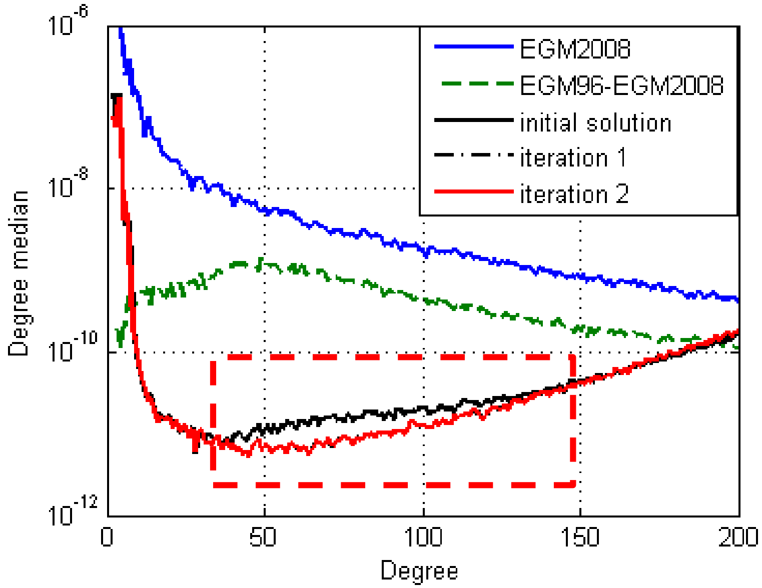
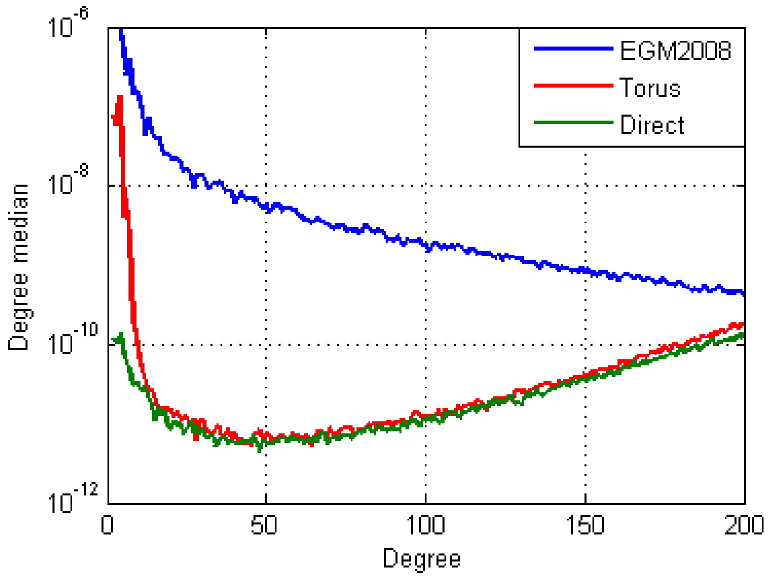
Torus with simulated satellite gradiometry data



Degree error median of models



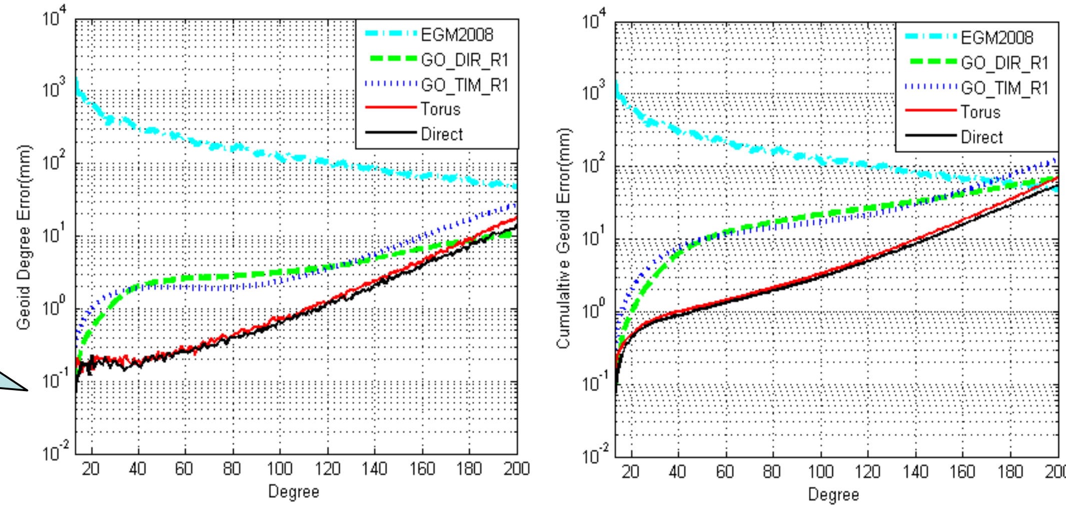
Degree error median of models from different method

After one iteration, the degree error of these coefficients 30<*L*<150 are better.

The model compiled by torus is slightly lower than direct.

Torus with simulated satellite gradiometry data

Degree and cumulative geoid error of models



The max Degree and cumulative geoid error

method / error (cm)	Torus	Direct
Geoid degree error	1.58	1.45
Cumulative geoid error	6.37	5.55

computational efficiency

method	Torus	Direct
CPU	1	106
Time spend (minute)	51	564

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Torus with real GOCE gradiometry data

- ✓ **Orbit Data**

  SST_PRD_2 (2009.11.1~2010.1.10, 71 days)

- ✓ **GOCE gradiometry observations (*Vxx* ,*Vyy and Vzz*)**

  EGG_NOM_2 (2009.11.1~2010.1.10, 71days)

- ✓ **reference model:**

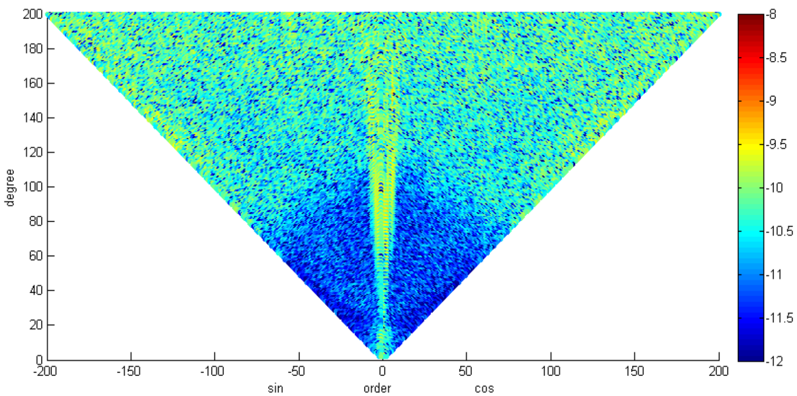
  EGM2008

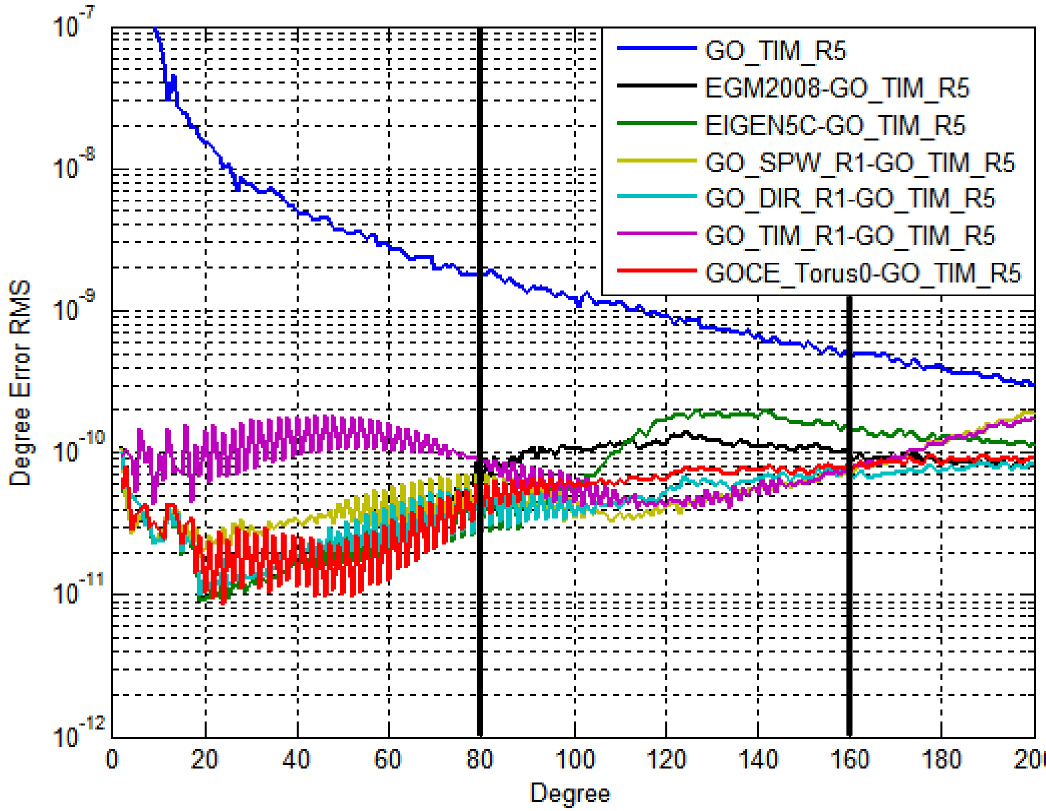
- ✓ **Filter: band-pass Butterworth and remove-restore approach**

- ✓ **Kaula's regularization technique**

Torus with real GOCE gradiometry data

Spectra of the geopotential coefficient differences between the Torus model and the GO_TIM_R5.





Degree RMS of the coefficient differences between different solutions and GO_TIM_R5

Torus with real GOCE gradiometry data

Validation of the different models up to d/o 200 using GPS-leveling data in USA (6169 points) (unit: m). The omission errors were disregarded.

Model	Mean	Max	Min	RMS	STD
GO_TIM_R5	-0.567	2.243	-3.056	0.765	0.513
EGM2008	-0.567	2.277	-3.046	0.766	0.516
GO_DIR_R1	-0.570	2.227	-3.018	0.768	0.515
GO_TIM_R1	-0.571	2.274	-3.029	0.775	0.524
GO_SPW_R1	-0.570	2.216	-2.960	0.777	0.528
GOCE_Torus0	-0.569	2.227	-3.038	0.771	0.521

Validation of the different models up to d/o 200 using GPS-leveling data in China (649 points) (unit: m). The omission errors were disregarded.

Model	Mean	Max	Min	RMS	STD
GO_TIM_R5	0.047	3.232	-3.007	0.570	0.569
EGM2008	0.047	3.831	-2.882	0.603	0.602
GO_DIR_R1	0.053	3.405	-3.134	0.577	0.575
GO_TIM_R1	0.048	3.345	-3.106	0.578	0.576
GO_SPW_R1	0.044	3.328	-3.062	0.576	0.575
GOCE_Torus0	0.048	3.422	-2.763	0.578	0.576

Torus with real GOCE gradiometry data

Validation of the different models using GPS-leveling data in China and USA(unit: m). The omission errors were compensated using the EGM2008 coefficients up to d/o 2190.

Model	Mean(USA)	STD(USA)	Mean(China)	STD(China)
GO_TIM_R5	-0.511	0.281	0.239	0.161
EGM2008	-0.511	0.284	0.239	0.240
GO_DIR_R1	-0.514	0.284	0.245	0.179
GO_TIM_R1	-0.515	0.295	0.240	0.191
GO_SPW_R1	-0.514	0.298	0.236	0.195
GOCE_Torus0	-0.513	0.289	0.240	0.194

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Conclusions

- ✓ **Torus models is revealed a similar accuracy with the models at the same period released by ESA.**
- ✓ **Fast resolution of gravity field based on massive amount of GOCE satellite gradiometry observations is feasible.**
- ✓ **The accuracy of GOCE_Torus0 is improved by 4.6 cm than EGM2008 corrected for the omission errors using the EGM2008 coefficients between the spherical harmonic degrees from 200 up to 2190.**

Outlooks

- ✓ **The high-degree and high precision gravity field model will be derived efficiently by torus from LL-SST data, HL-SST data and satellite gradiometry data.**
- ✓ **The torus approach will be expected to evaluate efficiently the performances of the next in-orbit satellite gravity missions.**

Thanks for your attention！

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Torus approach

$$V_{ij}\left(r,I,u,\Lambda\right)=\sum_{m=0}^{N}\sum_{k=-N}^{N}A_{mk}^{ij}\cos\psi_{mk}+B_{mk}^{ij}\sin\psi_{mk}$$

$$\begin{Bmatrix}A_{mk}^{ij}\\B_{mk}^{ij}\end{Bmatrix}=\sum_{n=n_{\min}[2]}^{N}H_{nmk}^{ij}\left(r,I\right)\begin{Bmatrix}\alpha_{nm}^{ij}\\\beta_{nm}^{ij}\end{Bmatrix}$$

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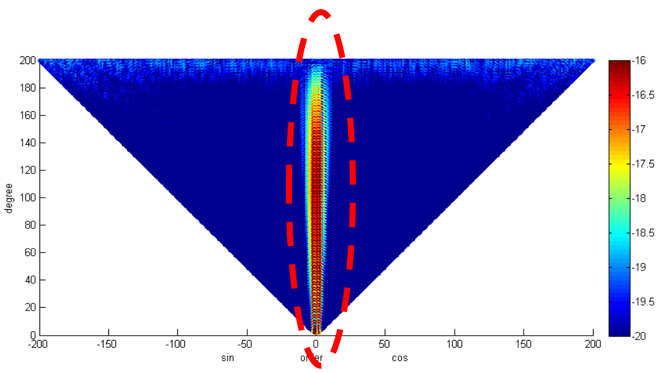
$$V_{yy}:H_{lmk}^{yy}=\frac{GM}{R^3}\left(\frac{R}{r}\right)^{l+3}\left[k^2-\left(l+1\right)^2\right]\bar{F}_{lmk}\left(I\right)$$

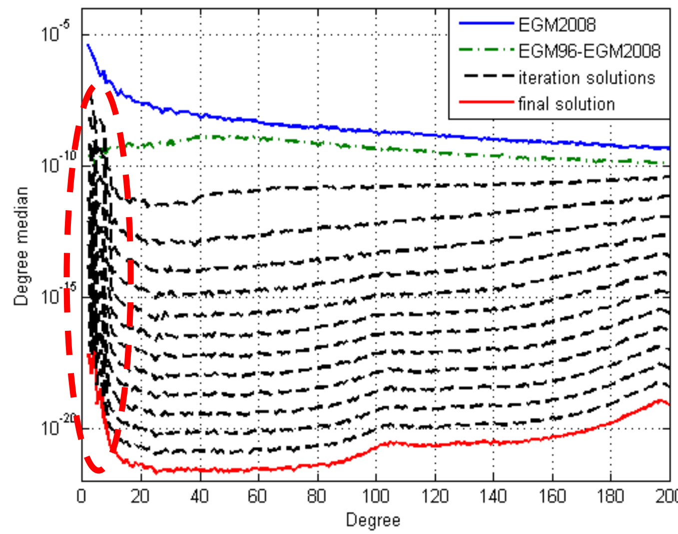
$$V_{zz}:H_{lmk}^{zz}=\frac{GM}{R^3}\left(\frac{R}{r}\right)^{l+3}\left[\left(l+1\right)\left(l+2\right)\right]\bar{F}_{lmk}\left(I\right)$$

example 2: torus with observations on orbits

以模型系数最大改正量小于10^{-14}作为迭代终止的条件，迭代13次仍未收敛，但此时除低阶（小于10）受极空白影响误差阶中值较大外，其余阶阶中值均在10^{-19}以内。



Torus解算模型相对于EGM2008的误差谱

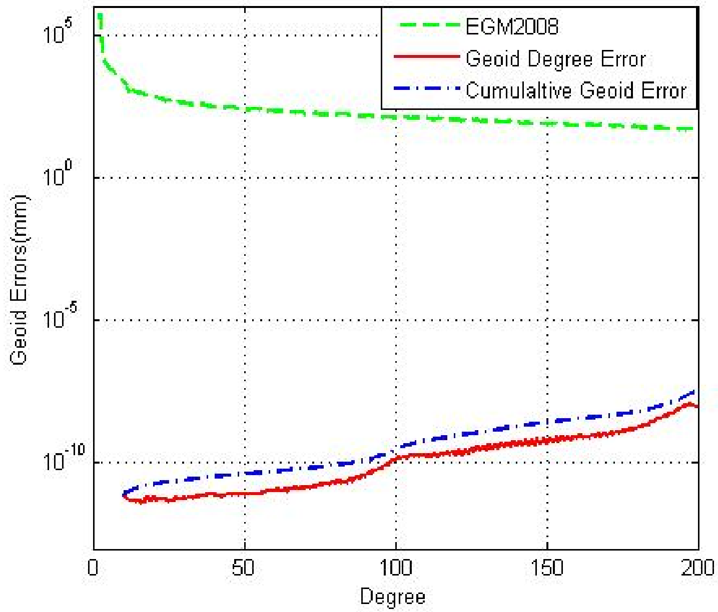


$$\sigma_n = median_m \left\{ \left| \bar{R}_{nm}^{est} - \bar{R}_{nm}^{ref} \right| \right\}$$

$$\bar{R}_{nm} = \left\{ \bar{C}_{nm}; \bar{S}_{nm} \right\}$$

相对于EGM2008的Torus模型误差阶中值

example 2: torus with observations on orbits



Torus模型相对于EGM2008的大地水准面阶误差和累积阶误差（未考虑m<10的系数）

大地水准面阶误差和累积阶误差计算公式：

$$\sigma_{N_n}=\left(\frac{\mu}{a\gamma}\right)\left(\frac{a}{R}\right)^{n+1}\sqrt{\sum_{m=m_{\min}}^{n}\left[\sigma_{\bar{C}_{nm}}^2+\sigma_{\bar{S}_{nm}}^2\right]}$$

$$\sigma_{N_n}^{(C)}=\sqrt{\sum_{i=2}^{n}\sigma_{N_i}^2}$$

200阶时大地水准面阶误差为8.48×10^{-9} mm，累积阶误差为3.05×10^{-8} mm

移去—恢复法的迭代策略可以进一步提高计算效率，将归算误差、格网化误差，以及参考模型的影响减小至可以忽略不计的程度。

In order to reduce the influence of low-precision components in coordinate system transformation, the simulation values are used to replace the low-precision components Vxy and Vyz. The effects of different filtering methods to deal with the colored noise in GOCE satellite gravitational gradient observations are compared and analyzed. The method combination Butterworth with remove-restore is proposed and verified by the GOCE satellite measured data.