The determination of earth's gravity field model by torus approach with GOCE data

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Introduction: significance of earth’s gravity field

geophysics
geodynamics
oceanography
global change
geodesy
Introduction: significance of earth’s gravity field

- geophysics
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Introduction: Gravity Field Exploration Missions

- CHAMP
- GRACE
- GOCE
GOCE overview

<table>
<thead>
<tr>
<th>role</th>
<th>Earth observation (EO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>orbit</td>
<td>Sun-synchronous ~224 km</td>
</tr>
<tr>
<td>Launch date</td>
<td>17 March 2009</td>
</tr>
<tr>
<td>Complete</td>
<td>11 Nov 2013</td>
</tr>
</tbody>
</table>

Mission objectives

✓ determine gravity-field anomalies with an accuracy of 1 mGal.
✓ determine the geoid with an accuracy of 1-2 cm.
✓ achieve the above at a spatial resolution better than 100 km.
Introduction: GOCE gravity field modeling

There are several different approaches applied to recover the GOCE gravity field.

Direct
time-wise
space-wise
SA
Tensor invariant
Rosborough
...

Introduction: GOCE gravity field modeling

There are several different approaches applied to recover the GOCE gravity field.

- Direct
- Time-wise
- Space-wise
- SA
- Tensor invariant method
- Rosborough

Approaches used to determine GOCE gravity field models by the HPF
Introduction: GOCE gravity field modeling

the max d/o is 200

40397 geopotential coefficients to be determined

tens of millions of SGG observations

Forming the normal equation and inverting the normal matrix will demand huge computation resources, which could not be realized using single processors.
Introduction: GOCE gravity field modeling

Torus approach

✓ combines the properties of space-wise and time-wise methods
✓ using the 2D-FFT and the block-diagonal least-squares adjustment

This method has been successfully applied to simulated data, but not used to compile the gravity field model with the real GOCE observations.
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Torus approach

Representation on the Sphere

\[ V_{ij}(r, \theta, \lambda) = \sum_{m=0}^{\infty} \left[ A_{m}^{ij}(r, \theta) \cos m\lambda + B_{m}^{ij}(r, \theta) \sin m\lambda \right] \]

Representation along the Orbit

\[ V_{ij}(r, I, u, \Lambda) = \sum_{m=0}^{N} \sum_{k=-N}^{N} A_{mk}^{ij} \cos \psi_{mk} + B_{mk}^{ij} \sin \psi_{mk} \]

\[
\begin{aligned}
A_{mk}^{ij} &= \sum_{n=n_{\text{min}}[2]}^{N} H_{nmk}^{ij}(r, I) \alpha_{nm}^{ij} \\
B_{mk}^{ij} &= \sum_{n=n_{\text{min}}[2]}^{N} H_{nmk}^{ij}(r, I) \beta_{nm}^{ij} 
\end{aligned}
\]

\[ \psi_{mk} = ku + m\Lambda \]

\[ u = u_{0} + \& \Delta t \]

\[ \Lambda = \Lambda_{0} + \& \Delta t \]

\( u \) is the argument of latitude

\( \Lambda \) is the longitude of ascending node
Torus approach

GOCE satellite orbits for one day (the red are ascending arcs, the blue are descending arcs)
Torus approach

Orbits on Torus
Torus approach

\[ V_{ij}(r, I, u, \Lambda) = \sum_{m=0}^{N} \sum_{k=-N}^{N} A_{mk}^{ij} \cos \psi_{mk} + B_{mk}^{ij} \sin \psi_{mk} \]

\[ \psi_{mk} = ku + m\Lambda \]

\[ u = u_0 + \upsilon \Delta t \quad \Lambda = \Lambda_0 + \alpha \Delta t \]

2D Fourier series

\[ V_{ij}(u, \Lambda) = \sum_{m=0}^{N} \sum_{k=-N}^{N} A_{mk}^{ij} \cos \psi_{mk} + B_{mk}^{ij} \sin \psi_{mk} \]

\[ \begin{aligned}
V_{xx} : H_{lmk}^{xx} &= \frac{GM}{R^3} \left( \frac{R}{r} \right)^{l+3} \left[ -\left( k^2 + l + 1 \right) \right] \bar{F}_{lmk}(I) \\
V_{yy} : H_{lmk}^{yy} &= \frac{GM}{R^3} \left( \frac{R}{r} \right)^{l+3} \left[ k^2 - (l + 1)^2 \right] \bar{F}_{lmk}(I) \\
V_{zz} : H_{lmk}^{zz} &= \frac{GM}{R^3} \left( \frac{R}{r} \right)^{l+3} \left[ (l + 1)(l + 2) \right] \bar{F}_{lmk}(I)
\end{aligned} \]
Torus approach-procedures

GOCE gradiometry data

Reference gravity field model

Observations no the nominal torus

Kriging

Observations at grids on the nominal torus

The difference between Observation and Reference

2D-FFT/IFFT

Lumped coefficient correction $\delta a^i$

Block diagonal least-squares adjustment

Spherical harmonic correction $\delta K^i$

New spherical harmonic coefficients $K^{i+1} = K^i + \delta K^i$

Converge?

No

Yes

Final solution of Spherical harmonic

Converge?

Yes

No
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Torus with simulated satellite gradiometry data

✓ Orbit Data
SST_PRD_2(2009.11.1~12.31, 61days),
sampling interval is 10s.
✓ gradiometry data (only $V_{zz}$)
observations are simulated on GOCE
real orbits using the model EGM2008,
the max d/o are 200.
✓ reference model: EGM96
✓ white noise 5mE/Hz$^{1/2}$

Simulated $V_{zz}$ observations
After one iteration, the degree error of these coefficients $30<L<150$ are better. The model compiled by torus is slightly lower than direct.
Torus with simulated satellite gradiometry data

Degree and cumulative geoid error of models

The max Degree and cumulative geoid error

<table>
<thead>
<tr>
<th>method</th>
<th>Torus</th>
<th>Direct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geoid degree error</td>
<td>1.58</td>
<td>1.45</td>
</tr>
<tr>
<td>Cumulative geoid error</td>
<td>6.37</td>
<td>5.55</td>
</tr>
</tbody>
</table>

Computational efficiency

<table>
<thead>
<tr>
<th>method</th>
<th>Torus</th>
<th>Direct</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU</td>
<td>1</td>
<td>106</td>
</tr>
<tr>
<td>Time spend (minute)</td>
<td>51</td>
<td>564</td>
</tr>
</tbody>
</table>

The graph shows the comparison of Geoid Degree Error and Cumulative Geoid Error for different methods.
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✓ Orbit Data

SST_PRD_2 (2009.11.1~2010.1.10, 71 days)

✓ GOCE gradiometry observations (Vxx, Vyy and Vzz)

EGG_NOM_2 (2009.11.1~2010.1.10, 71 days)

✓ reference model:

EGM2008

✓ Filter: band-pass Butterworth and remove-restore approach

✓ Kaula's regularization technique
Torus with real GOCE gradiometry data

Spectra of the geopotential coefficient differences between the Torus model and the GO_TIM_R5.

Degree RMS of the coefficient differences between different solutions and GO_TIM_R5
Torus with real GOCE gradiometry data

Validation of the different models up to d/o 200 using GPS-leveling data in USA (6169 points) (unit: m). The omission errors were disregarded.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO_TIM_R5</td>
<td>-0.567</td>
<td>2.243</td>
<td>-3.056</td>
<td>0.765</td>
<td>0.513</td>
</tr>
<tr>
<td>EGM2008</td>
<td>-0.567</td>
<td>2.277</td>
<td>-3.046</td>
<td>0.766</td>
<td>0.516</td>
</tr>
<tr>
<td>GO_DIR_R1</td>
<td>-0.570</td>
<td>2.227</td>
<td>-3.018</td>
<td>0.768</td>
<td>0.515</td>
</tr>
<tr>
<td>GO_TIM_R1</td>
<td>-0.571</td>
<td>2.274</td>
<td>-3.029</td>
<td>0.775</td>
<td>0.524</td>
</tr>
<tr>
<td>GO_SPW_R1</td>
<td>-0.570</td>
<td>2.216</td>
<td>-2.960</td>
<td>0.777</td>
<td>0.528</td>
</tr>
<tr>
<td>GOCE_Torus0</td>
<td>-0.569</td>
<td>2.227</td>
<td>-3.038</td>
<td>0.771</td>
<td>0.521</td>
</tr>
</tbody>
</table>

Validation of the different models up to d/o 200 using GPS-leveling data in China (649 points) (unit: m). The omission errors were disregarded.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO_TIM_R5</td>
<td>0.047</td>
<td>3.232</td>
<td>-3.007</td>
<td>0.570</td>
<td>0.569</td>
</tr>
<tr>
<td>EGM2008</td>
<td>0.047</td>
<td>3.831</td>
<td>-2.882</td>
<td>0.603</td>
<td>0.602</td>
</tr>
<tr>
<td>GO_DIR_R1</td>
<td>0.053</td>
<td>3.405</td>
<td>-3.134</td>
<td>0.577</td>
<td>0.575</td>
</tr>
<tr>
<td>GO_TIM_R1</td>
<td>0.048</td>
<td>3.345</td>
<td>-3.106</td>
<td>0.578</td>
<td>0.576</td>
</tr>
<tr>
<td>GO_SPW_R1</td>
<td>0.044</td>
<td>3.328</td>
<td>-3.062</td>
<td>0.576</td>
<td>0.575</td>
</tr>
<tr>
<td>GOCE_Torus0</td>
<td>0.048</td>
<td>3.422</td>
<td>-2.763</td>
<td>0.578</td>
<td>0.576</td>
</tr>
</tbody>
</table>
Validation of the different models using GPS-leveling data in China and USA (unit: m). The omission errors were compensated using the EGM2008 coefficients up to d/o 2190.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean(USA)</th>
<th>STD(USA)</th>
<th>Mean(China)</th>
<th>STD(China)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO_TIM_R5</td>
<td>-0.511</td>
<td>0.281</td>
<td>0.239</td>
<td>0.161</td>
</tr>
<tr>
<td>EGM2008</td>
<td>-0.511</td>
<td>0.284</td>
<td>0.239</td>
<td>0.240</td>
</tr>
<tr>
<td>GO_DIR_R1</td>
<td>-0.514</td>
<td>0.284</td>
<td>0.245</td>
<td>0.179</td>
</tr>
<tr>
<td>GO_TIM_R1</td>
<td>-0.515</td>
<td>0.295</td>
<td>0.240</td>
<td>0.191</td>
</tr>
<tr>
<td>GO_SPW_R1</td>
<td>-0.514</td>
<td>0.298</td>
<td>0.236</td>
<td>0.195</td>
</tr>
<tr>
<td>GOCE_Torus0</td>
<td>-0.513</td>
<td>0.289</td>
<td>0.240</td>
<td>0.194</td>
</tr>
</tbody>
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Conclusions

✓ Torus models is revealed a similar accuracy with the models at the same period released by ESA.

✓ Fast resolution of gravity field based on massive amount of GOCE satellite gradiometry observations is feasible.

✓ The accuracy of GOCE_Torus0 is improved by 4.6 cm than EGM2008 corrected for the omission errors using the EGM2008 coefficients between the spherical harmonic degrees from 200 up to 2190.
Outlooks

✓ The high-degree and high precision gravity field model will be derived efficiently by torus from LL-SST data, HL-SST data and satellite gradiometry data.

✓ The torus approach will be expected to evaluate efficiently the performances of the next in-orbit satellite gravity missions.
Thanks for your attention!

Email: liuhl@casm.ac.cn
Torus approach

\[ V_{ij}(r, I, u, \Lambda) = \sum_{m=0}^{N} \sum_{k=-N}^{N} A_{mk}^{ij} \cos \psi_{mk} + B_{mk}^{ij} \sin \psi_{mk} \]

\[ \psi_{mk} = ku + m\Lambda \]

\[ u = u_0 + i\Delta t \]

\[ \Lambda = \Lambda_0 + \beta \Delta t \]

2D Fourier series

\[ V_{ij}(u, \Lambda) = \sum_{m=0}^{N} \sum_{k=-N}^{N} A_{mk}^{ij} \cos \psi_{mk} + B_{mk}^{ij} \sin \psi_{mk} \]

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\[ V_{yy} : H_{lmk}^{yy} = \frac{GM}{R^3} \left( \frac{R}{r} \right)^{l+3} \left[ k^2 - (l + 1)^2 \right] \bar{F}_{lmk}(I) \]

\[ V_{zz} : H_{lmk}^{zz} = \frac{GM}{R^3} \left( \frac{R}{r} \right)^{l+3} \left[ (l + 1)(l + 2) \right] \bar{F}_{lmk}(I) \]
example 2: torus with observations on orbits

以模型系数最大改正量小于 $10^{-14}$ 作为迭代终止的条件，迭代13次仍未收敛，但此时除低阶（小于10）受极空白影响误差阶中值较大外，其余阶中值均在 $10^{-19}$ 以内。

\[ \sigma_n = \text{median}_m \{ | \overline{R}_{nm}^{\text{est}} - \overline{R}_{nm}^{\text{ref}} | \} \]

\[ \overline{R}_{nm} = \{ \overline{C}_{nm}; \overline{S}_{nm} \} \]
example 2: torus with observations on orbits

Torus模型相对于EGM2008的大地水准面阶误差和累积阶误差计算公式:

\[
\sigma_{N_n} = \left( \frac{\mu}{a\gamma} \right) \left( \frac{a}{R} \right)^{n+1} \sqrt{\sum_{m=m_{\text{min}}}^{n} \left( \sigma_{C_{nm}}^2 + \sigma_{\xi_{nm}}^2 \right)}
\]

\[
\sigma_{N_n}^{(C)} = \sqrt{\sum_{i=2}^{n} \sigma_{N_i}^2}
\]

200阶时大地水准面阶误差为8.48×10⁻⁹ mm，累积阶误差为3.05×10⁻⁸ mm

移去—恢复法的迭代策略可以进一步提高计算效率，将归算误差、格网化误差，以及参考模型的影响减小至可以忽略不计的程度。
In order to reduce the influence of low-precision components in coordinate system transformation, the simulation values are used to replace the low-precision components \( V_{xy} \) and \( V_{yz} \). The effects of different filtering methods to deal with the colored noise in GOCE satellite gravitational gradient observations are compared and analyzed. The method combination Butterworth with remove-restore is proposed and verified by the GOCE satellite measured data.